

1. **Problem**

Compute the Hessian of the function

$$f(x_1, x_2) = 7x_1^2 + 5x_1x_2 + 3x_2^2$$

at  $(x_1, x_2) = (1, 4)$ . What is the value of the upper left element?

- (a) 6
- (b) 7
- (c) 14
- (d) 5
- (e) -19

**Solution**

The first-order partial derivatives are

$$\begin{aligned}f_1'(x_1, x_2) &= 14x_1 + 5x_2 \\f_2'(x_1, x_2) &= 5x_1 + 6x_2\end{aligned}$$

and the second-order partial derivatives are

$$\begin{aligned}f_{11}''(x_1, x_2) &= 14 \\f_{12}''(x_1, x_2) &= 5 \\f_{21}''(x_1, x_2) &= 5 \\f_{22}''(x_1, x_2) &= 6\end{aligned}$$

Therefore the Hessian is

$$f''(x_1, x_2) = \begin{pmatrix} 14 & 5 \\ 5 & 6 \end{pmatrix}$$

independent of  $x_1$  and  $x_2$ . Thus, the upper left element is:  $f_{11}''(1, 4) = 14$ .

- (a) False
- (b) False
- (c) True
- (d) False
- (e) False