1. **Problem**
Compute the Hessian of the function

$$f(x_1, x_2) = 7x_1^2 + 5x_1x_2 + 3x_2^2$$

at $(x_1, x_2) = (1, 4)$. What is the value of the upper left element?

(a) 6

(b) 7

(c) 14

(d) 5

(e) -19

Solution
The first-order partial derivatives are

$$f_1'(x_1, x_2) = 14x_1 + 5x_2$$
$$f_2'(x_1, x_2) = 5x_1 + 6x_2$$

and the second-order partial derivatives are

$$f_{11}''(x_1, x_2) = 14$$
$$f_{12}''(x_1, x_2) = 5$$
$$f_{21}''(x_1, x_2) = 5$$
$$f_{22}''(x_1, x_2) = 6$$

Therefore the Hessian is

$$f''(x_1, x_2) = \begin{pmatrix} 14 & 5 \\ 5 & 6 \end{pmatrix}$$

independent of x_1 and x_2. Thus, the upper left element is: $f_{11}''(1, 4) = 14$.

(a) False

(b) False

(c) True

(d) False

(e) False