

### 1. Problem

For the matrix

$$A = \begin{pmatrix} 16 & 4 & 16 & -4 \\ 4 & 5 & 6 & -9 \\ 16 & 6 & 33 & -28 \\ -4 & -9 & -28 & 58 \end{pmatrix}.$$

compute the matrix  $L = (\ell_{ij})_{1 \leq i, j \leq 4}$  from the Cholesky decomposition  $A = LL^\top$ .

Which of the following statements are true?

- (a)  $\ell_{41} = -1$
- (b)  $\ell_{44} < 4$
- (c)  $\ell_{22} = -1$
- (d)  $\ell_{11} > 0$
- (e)  $\ell_{32} \leq 1$

### Solution

The decomposition yields

$$L = \begin{pmatrix} 4 & 0 & 0 & 0 \\ 1 & 2 & 0 & 0 \\ 4 & 1 & 4 & 0 \\ -1 & -4 & -5 & 4 \end{pmatrix}$$

and hence:

- (a) True.  $\ell_{41} = -1$
- (b) False.  $\ell_{44} = 4 \not< 4$
- (c) False.  $\ell_{22} = 2 \neq -1$
- (d) True.  $\ell_{11} = 4$
- (e) True.  $\ell_{32} = 1$